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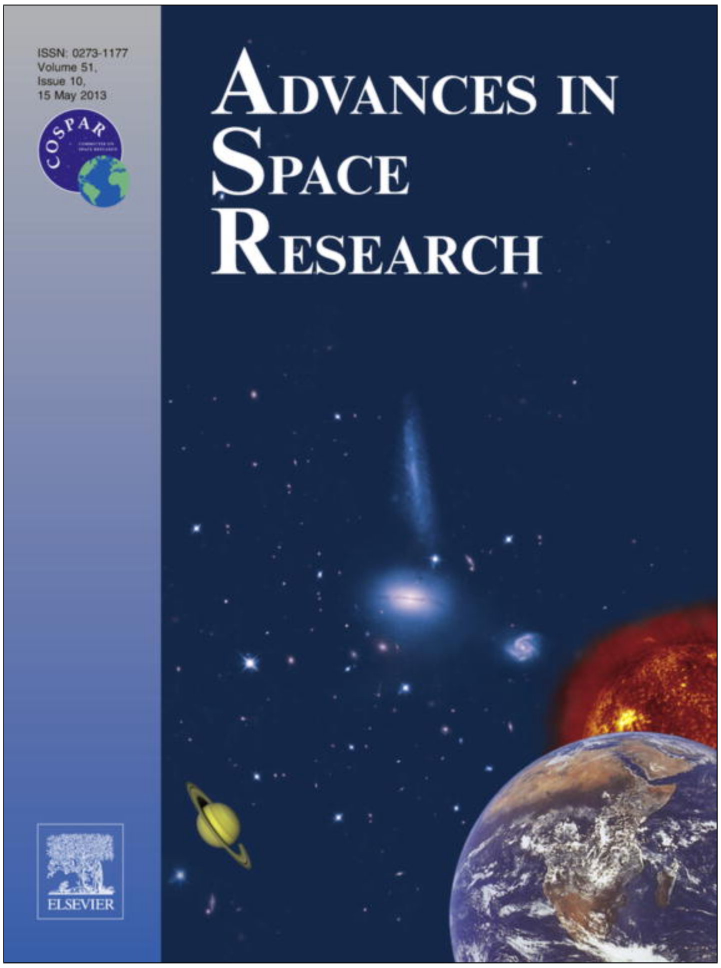

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Semi-empirical model of the maximum electron concentration in the ionosphere: Comparison with data from Toluca (México)

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Abstract

A simple semi-empirical model to determine the maximum electron concentration in the ionosphere (N_mF_2) for South American locations is used to calculate N_mF_2 for a northern hemisphere station in the same longitude sector. N_mF_2 is determined as the sum of two terms, one related to photochemical and diffusive processes and the other one to transport mechanisms. The model gives diurnal variations of N_mF_2 representative for winter, summer and equinox conditions, during intervals of high and low solar activity. Model N_mF_2 results are compared with ionosonde observations made at Toluca-México (19.3°N; 260°E). Differences between model results and observations are similar to those corresponding to comparisons with South American observations. It seems that further improvement of the model could be made by refining the latitude dependencies of coefficients used for the transport term.

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Keywords: Ionosphere; Critical frequency; Semi-empirical model

1. Introduction

A wide variety of models to determine the maximum electron concentration in the ionosphere (N_mF_2), or the critical frequency of the F-region as observed by an ionosonde (f_oF_2), have been proposed over several decades. Both physical and empirical models spanning a large range of complexity have been devised for regional and global use. Five comprehensive physical and well known models have been inter-compared and tested with observations (Anderson et al., 1998). Two of these are self-consistent and the others rely on empirical representations of atmospheric conditions including thermospheric winds. On the statistical side, even today global representations of f_oF_2 in terms of series of spherical harmonic functions are used in models such as the International Reference Ionosphere (IRI), which is regularly up-dated (Bilitza et al., 2011). Fairly recently, alternative representations of global observations of f_oF_2 using neural networks training (McKinnell and Oyeyemi, 2009) or non-linear fitting techniques (Hoque et al., 2011) have also been developed.

Nevertheless, single station or regional simple modeling of f_oF_2 seems to be required for some purposes (Liu et al., 2012). It is along this line that a semi-empirical model for N_mF_2 over South American middle latitudes was developed two decades ago (Arriagada and Foppiano, 1992). The model is based on simple chemical and physical processes as suggested by Rishbeth (1986). Here, the model is run to determine N_mF_2 over Toluca-México (19.3°N; 260°E) for a range of geophysical conditions. Then, model N_mF_2 results converted into f_oF_2 values are contrasted with ionosonde observations of f_oF_2 made over several years at the same location (Cipagauta, 2007).

2. Semi-empirical model

Values of N_mF_2 are determined as the sum of two terms: one associated with photochemical-diffusive processes and

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the other is a drag-transport related term. Detailed equations are given in the Appendix.

The first term is based on the model originally proposed by Rishbeth (1967), in which electron density, N_m, for a volume element located at the level where the electron concentration is maximum, is determined by the continuity equation:

$$\frac{dN_m}{dt} = q - L_Q - L_D \tag{1}$$

where q is the rate of production, L_Q is the loss rate of electron concentration by chemical reactions and L_D is the loss rate of electron concentration by vertical diffusion only. For q the Chapman production function is adopted (Chapman, 1931a,b). Both electron loss terms are assumed to be proportional to electron concentration and the proportionality coefficients to decrease exponentially with height. These losses approximation is based on standard theory (Rishbeth & Garriot, 1969): N_mF_2 occurs at a level where diffusion and chemical loss are of comparable importance.

Table 1
Months and years used to determine the seasonal mean values of critical frequencies in the region F of the ionosphere, f_0F_2, for Toluca-México (19.3°N; 260°E).

	Low solar activity	High solar activity
Winter	November 1973, 1974, 1975, 1976	November 1979, 1980, 1981
	December 1973, 1974, 1975, 1976	December 1979, 1980, 1981
	January 1973, 1974, 1975, 1976	January 1979, 1980, 1981
	February 1973, 1974, 1975, 1976	February 1979, 1980, 1981
Equinox	March 1973, 1974, 1975, 1976	March 1979, 1980, 1981
	April 1973, 1974, 1975, 1976	April 1979, 1980, 1981
	September 1974, 1975, 1976	September 1979, 1980, 1981
	October 1973, 1974, 1975, 1976	October 1979, 1980, 1981
Summer	May 1973, 1974, 1975, 1976	May 1979, 1980, 1981
	June 1973, 1974, 1975, 1976	June 1979, 1980, 1981
	July 1973, 1974, 1975, 1976	June 1979, 1980, 1981
	August 1973, 1974, 1975, 1976	August 1979, 1980, 1981

The transport term is simply a harmonic series of two components: one diurnal and the other semi-diurnal. The

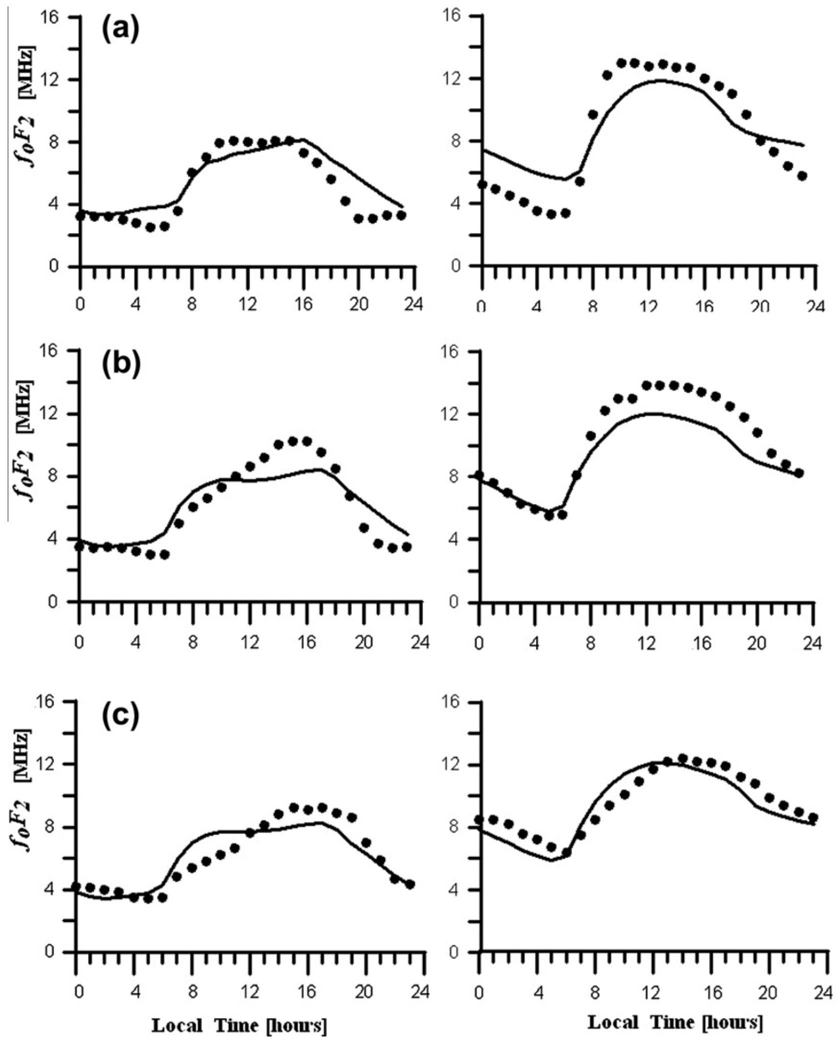

Fig. 1. Diurnal variation of observed (filled circles) seasonal median $f_oF_2[MHz]$ for Toluca-México (19.3°N; 260°E) and those calculated with the semi-empirical model (full line). (left) Low solar activity level. (right) High solar activity level. (a) Winter. (b) Equinox. (c) Summer.

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Table 2
Amplitude coefficients and phase angles for transport term.

	Winter		Equinox		Summer	
	Model	Local	Model	Local	Model	Local
Low solar activity						
C_0	3.15	4.5	3.13	5.4	3.12	5.7
C_1	−0.16	1.3	−0.16	1.7	−0.16	1.3
C_2	0.5	0.3	0.5	0.9	0.5	0.7
$\varphi_1(rad)$	0.56	−0.17	0.56	0.58	0.56	1.47
$\varphi_2(rad)$	3.38	0.17	3.51	−1.27	3.64	−0.49
High solar activity						
C_0	6.31	10.5	6.27	12.9	6.44	11.8
C_1	−0.16	2.7	−0.16	1.8	−0.16	0.9
C_2	0.5	0.8	0.5	0.5	0.5	0.4
$\varphi_1(rad)$	1.23	0.47	1.23	0.81	1.23	−1.24
$\varphi_2(rad)$	0.38	−1.21	0.4	−0.74	0.41	1.17

coefficients are calculated using functions that depend on latitude, solar declination and sunspot number. These functions were derived from coefficients first determined for a few locations in South America. At each location it was assumed that the differences between observed N_mF_2 and Nm calculated by the photochemical-diffusive processes are very well described by an only two Fourier components model (Arriagada, 1988; Burgos 2002 private communication).

3. Results

Fig. 1 shows model results in terms of f_oF_2 for three seasons and two levels of solar activity. Main differences among the diurnal variations are associated with solar activity level rather than with season. For low solar activity, the diurnal variations are characterized by an afternoon maximum and a sort of secondary morning maximum. By contrast, the diurnal variations exhibit a single maximum just after noon during high solar activity.

Fig. 1 also shows seasonal mean f_oF_2 derived from observations made at Toluca-México (19.3°N; 260°E). The intervals used for these computations are given in Table 1. Model f_oF_2 diurnal variations agree fairly well

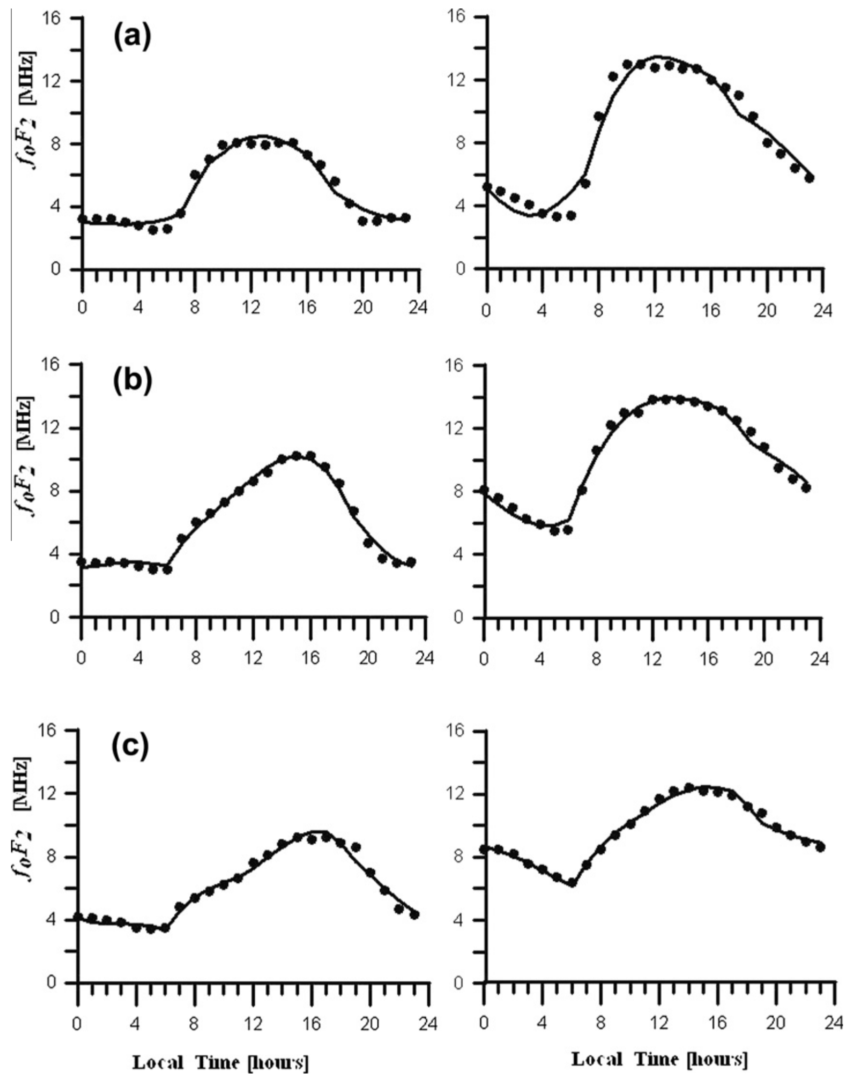

Fig. 2. Same as Fig. 1 but for locally fitted semi-empirical model.

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Table 3
Root mean square error for model and local fitting.

Location	RMS error	Low solar activity			High solar activity		
		Winter	Equinox	Summer	Winter	Equinox	Summer
Toluca 19.3°N; 260°E	Model	0.96	1.43	0.97	1.76	1.41	0.81
	Local	0.39	0.29	0.27	0.69	0.35	0.22
Concepción 36.8°S; 73.0°W	Model	2.03	1.23	0.99	2.01	1.06	0.93
	Local	0.39	0.29	0.27	0.48	0.36	0.19

with observed variations for winter at low solar activity and summer at high solar activity. Although the agreement is not good for winter and equinox during high solar activity, at least the shape of model and observed diurnal variations are similar. The shapes of model diurnal variations do not agree with the observed ones for equinox and summer during low solar activity. However, overall results are similar to those obtained for Concepción (36.8°S; 73.0°W) and King George Island (62.2°S; 68.8°W) for the same geophysical conditions (Arriagada and Foppiano, 1992). Table 3 gives the root mean square error (labeled Model) for Toluca and Concepción.

As an alternative procedure, a transport related term (Eq. A.3) could be determined by computing local amplitude coefficients and phase angles, i.e. calculating the diurnal and semi-diurnal Fourier components of the differences between observed N_mF_2 for Toluca-México, and the photochemical-diffusive N_mF_2 term. Fig. 2 shows model and observed f_oF_2 for such a case. The amplitude coefficients and phase angles used are given in Table 2. Obviously, the agreement between model and observed variations is almost perfect. It is suggested that the transport term is indeed very well represented by only a diurnal and a semi-diurnal component.

Even a third approach would be possible. To derive new functions for the latitudinal dependencies of the coefficients and phase angles using the values of Table 2 (labeled Local), so as to replace the functions given by Eqs. A.4.

4. Conclusions

N_mF_2 is determined for a northern hemisphere location using a very simple semi-empirical model derived for South American locations. The corresponding model f_oF_2 diurnal variations are found to account for observed diurnal variations at Toluca-México (19.3°N; 260°E) with somewhat similar degree of agreement than that found for South American locations, except for winter during low solar activity. Moreover, the agreement for the local case is almost perfect (see Table 3).

The model's transport related term may be improved so as to better represent northern hemisphere conditions by using results from the application of the model to observations made at Toluca-México.

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Appendix A. Semi-empirical model of N_mF_2 for South American middle latitudes

The maximum electron concentration of the ionosphere, N_mF_2, is determined following the scheme proposed by Arriagada and Foppiano (1992) as: $N_mF_2 = N_m + \Delta N_m$, where N_m is a photochemical-diffusive term and ΔN_m is a transport related term. N_m is determined by numerically solving the continuity equation for the normalized height of maximum electron concentration, z_m,

$$\frac{dN_m}{dt} = q_0 \exp(1 - z_m - \exp(-z_m)Ch(x,\chi)) - c_N N_m \beta_0 \exp(-Kz_m) \tag{A.1}$$

where q_0 is the rate of production for vertical incidence ionization at the height of peak production, which is assumed to depend only on the solar activity level as

$$q_0 = 902.775(1 + 0.0054R) \tag{A.2}$$

R is the twelve-month running mean international sunspot number, $Ch(x,\chi)$ is the grazing incidence Chapman function for a spherically stratified atmosphere. $x = (R_E + h)/H$, where R_E is the Earth's radius, h is height and H is scale height, χ is the solar zenith angle. z_m is taken as

Sunrise: $z_{m1} = \ln Ch(x,\chi) + c_{z1}$ (Rishbeth, 1967)
Daytime: $z_{m2} = \frac{1}{K+1}\ln\left(\frac{\beta_0}{d_0 L_e}\right)$ (Rishbeth and Barron, 1960)
Nighttime: $z_{m3} = \frac{1}{K+1}\ln\left(\frac{\beta_0}{d_0 L_s}\right)$ (Dungey, 1956)

Values adopted for constants are $K = 1.75$; $\beta_0 = 1[10^{-2}s^{-1}]$; $d_0 = 2.5[10^{-5}s^{-1}]$; $L_e = 0.8$; $L_s = 0.15$; sunrise $c_N = 1.25$; day $c_N = 1.25$; night $c_N = 1.6$ and $c_{z1} = 0.25$, as quoted by Rishbeth (1967).

The transport related term is determined as

$$\Delta N_m = 1.24 \times 10^4 \left[C_0 + C_1 \cos\left(\frac{2\pi}{24}(t - 3.18\varphi_1)\right) + C_2 \cos\left(\frac{4\pi}{24}(t - 1.9\varphi_2)\right)\right], \tag{A.3}$$

where the amplitude coefficients and phase angles are

$$\begin{aligned} C_0 &= 2.39(1 - 0.024X)(1 - 0.017\delta)(1 + 0.0167R) \\ C_1 &= 0.2(0.307X - 1)(1 - 0.012\delta) \\ C_2 &= 0.5(1 + 0.026\delta) \\ \varphi_1 &= 0.39(1 + 0.0169\delta)(1 + 0.0216R) \\ \varphi_2 &= 9.09(1 - 0.0144X)(1 + 0.041\delta)(1 + 0.0044R). \end{aligned} \tag{A.4}$$

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X is the modified magnetic dip of the location ($X = \tan^{-1}(I/\sqrt{\cos\lambda})$; I magnetic dip, λ geographic latitude and δ is the solar declination.

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